NEW ASPECTS CONCERNING THE DEFINITION OF MICROSTRIP CHARACTERISTIC IMPEDANCE AS A FUNCTION OF FREQUENCY

by Rolf H. Jansen and Norbert H.L. Koster

Duisburg University, Department of Electrical Engineering, FB9/ATE
Bismarckstraße 81, D-41oo Duisburg 1, W.Germany
Rolf H. Jansen is presently with Standard Elektrik Lorenz AG, CP/EFR
Ostendstraße 3, D-753o Pforzheim, W.Germany

ABSTRACT

The definition of microstrip characteristic impedance is considered in conjunction with a rigorous frequency dependent hybrid-mode approach to planar n-ports. Analytical reflections of general validity and numerical results obtained for the nonsymmetrical gap and the microstrip impedance step provide new aspects for a proper choice of the definition.

INTRODUCTION

The question of the frequency dependence of microstrip characteristic impedance is of considerable interest for microstrip circuit design at higher microwave and at millimeter-wave frequencies. This is mirrored by the contributions of several authors who have dealt with the problem in the past years, see for example refs.1-5, however, a completely satisfying answer to this question has not been given to the present time[5,6]. Nevertheless, with a number of rigorous hybrid-mode microstrip solutions being available by now[7,8], it should be commonly accepted if the discussion is restricted to these and if characteristic impedance is treated as a property of microstrip itself, i.e. aspects of launching are excluded. Under these presumptions there are three preferred candidates for a reasonable definition of frequency dependent microstrip characteristic impedance, namely

$$Z_o^I=2P/|I|^2, \quad (1a)$$

$$\bar{Z}_o=U/I \quad , \quad (1b)$$

$$Z_o^U=|U|^2/2P, \quad (1c)$$

with the meaning of the symbols used in analogy to those of ref.7. I denotes the longitudinal strip current, U the strip center voltage and P the power transported by the fundamental microstrip mode. P is evaluated from the transverse electromagnetic field of microstrip according to Pointing's theorem. Among the above three definitions, (1a) is that which exhibits the smallest variation with frequency in the normal range of microstrip applications. Numerical results obtained for the voltage/current expression (1b) show a moderate increase with frequency which is about 10 % for a strip of 5o Ohms on a standard alumina substrate in the 0-16 GHz range[8]. For the characteristic impedance defined by (1c) the increase with frequency is approximately doubled. The characteristic impedance of (1b) is the geometrical mean of those according to (1a) and (1c). It should be noted further that the use of (1a) offers some advantage for a unified treatment of strip and slot transmission lines[9].

The results presented here have been elaborated in conjunction with a rigorous frequency dependent spectral-domain approach to the planar n-port problem which was recently developed by Jansen[10]. With such an instrument available for the analysis of planar structures like microstrip, slot and coplanar discontinuities, the problem of the definition of characteristic impedance in the case of hybrid-mode waveguides has to be illuminated anew. The question arises, in which way network quantities, like for example scattering and impedance matrices, have to be derived from the numerically computed electromagnetic field of planar configurations such that this promises the most accurate circuit representation and is best suited for CAD purposes. The difficulty is, that the general microwave network approach, i.e. the hollow waveguide concept, which is based on wave-impedance and can be extended to total characteristic impedance in the special case of TEM lines[11,12], is not applicable to microstrip rigorously, except for zero operating frequency. In contrast to hollow waveguides, the transverse electromagnetic field distributions of microstrip modes change with increasing frequency. The fields of microstrip lines in a planar circuit cannot be separated strictly from each other. With the exception of purely transverse discontinuities, microstrip junctions cannot be treated rigorously in terms of complete modal field expansions on the encountered lines. Furthermore, even if the numerical computation of the electromagnetic field of simple microstrip circuits as a whole seems to be feasible with the approach used here, CAD of this kind would be far beyond the capability of present computers. Also, this would not yet be able to include active elements into the circuits considered and thus be of limited usefulness only. With the present state of the art, a network concept based on the segmentation of microstrip and related planar circuits is an unavoidable necessity for the application of CAD. A concept used for microstrip circuits should preferably be of the quasi-TEM type in view of easy compatibility with active and passive lumped elements and with existing measurement techniques. So, the question is primarily, which of the characteristic impedance formulations (1a), (1b) or (1c) is best suited to define network analogs of microstrip structures, particularly scattering and impedance matrices, with properties in good agreement with the properties of the associated electromagnetic field. Besides, conceptual advantages and the practicability for CAD purposes have to be considered.

GENERAL CONSIDERATIONS

In the expressions (1a) to (1c) of microstrip characteristic impedance three physical quantities are employed, namely I, U and P, which are related to the hybrid-mode electromagnetic field in a definite way independent of frequency. Only two of these can be chosen independently in a microstrip network concept, the third one is compatible with the two others only at zero frequency. The power P associated with a wave on microstrip and the longitudinal strip current I are uniquely defined integral quantities, whereas there is some arbitraryness in the choice of the integration path used for the derivation of U from the transverse electric field. The hybrid-mode electromagnetic field of microstrip contains both a longitudinal electric and a longitudinal magnetic component. Therefore, neither the current I nor the voltage U can describe

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completely the associated transverse magnetic field and transverse electric field, respectively. To illustrate this clearly, consider the spectral-domain relationships

$$j_{xn}=Y_{an}e_{xn}+Y_{bn}e_{yn}$$
$$j_{yn}=Y_{bn}e_{xn}+Y_{cn}e_{yn} \qquad (2)$$

linking the surface current density j and the electric field e of a microstrip line in the plane of the substrate surface[8]. This shows that the transverse x-components and the longitudinal y-components are all coupled among each other and cannot be described separately. As a consequence, it can be concluded that in a microstrip network concept based on the quantities I and U the power properties of microstrip n-ports cannot be described correctly at high frequencies. In other words, if from the electromagnetic field of a lossless microstrip junction the quantities I and U are derived and are used to construct a network matrix, the latter cannot be expected to have the corresponding power properties. Finally, if the wave amplitude $\bar{a}$ of a microstrip wave propagating at higher frequencies is calculated from I and U applying the relationships of the waveguide concept[1,2] it is found that

$$\frac{1}{2}|\bar{a}|^2=\frac{1}{2}|I|^2\bar{Z}_o=\frac{1}{2}|U|^2/\bar{Z}_o \neq P. \qquad (3)$$

So, the voltage/current definition (1b) of microstrip characteristic impedance does not satisfy one of the fundamental equations of the waveguide concept of microwave circuits. On the other hand, wave amplitudes a^I and a^U computed on the basis of the definitions (1a) and (1c) do exhibit the direct relationship to the transported power P, since

$$\frac{1}{2}|a^I|^2=\frac{1}{2}|I|^2Z_o^I = P,$$
$$\frac{1}{2}|a^U|^2=\frac{1}{2}|U|^2/Z_o^U = P \qquad (4)$$

holds. So, from the point of view considered, expressions (1a) and (1c) are potentially suited for a microstrip network concept whereas the definition (1b) can be excluded a priori. Eq. (4) implies that the power properties of the electromagnetic field of a microstrip n-port are automatically conserved in the construction of scattering matrices for the wave amplitudes a^I and a^U if the reference planes are chosen in consistence with the waveguide concept. It is obvious that this would also hold for other definitions of U, i.e. with integration paths in the transverse plane of microstrip but deviating from the strip center line.

SPECIAL CONSIDERATIONS

In the following it is outlined in which way network matrices are derived from the hybrid-mode field of planar structures in conjunction with the numerical approach[10] used by the authors. This throws additional light on the problem under discussion. In the approach applied, the microstrip n-port investigated is thought to be operated under resonance conditions in a number of n fictitious experiments where n is equal to the number of ports. In each experiment, resonance is induced by proper choice of the length of one out of n ideally short-circuited stubs (position of the respective shielding wall) being attached to the reference planes of the n-port. As an example, consider the case of a microstrip impedance step in fig. 1 and the notations indicated there.

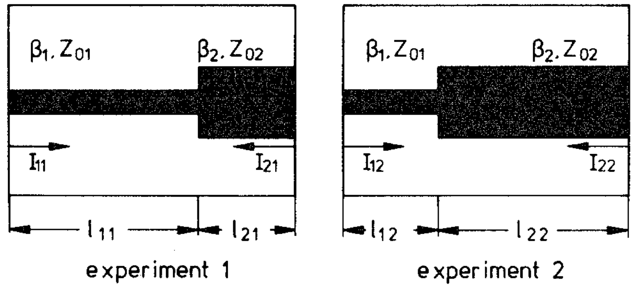

Fig. 1: Illustration of fictitious resonance experiments for a microstrip impedance step

The numerical simulation of the experiments is performed by computing the associated eigensolutions and three-dimensional fields from which the information required for the construction of the n-port scattering matrix is extracted. It has been described in the appendix of ref.10 how this is achieved. In terms of the longitudinal strip current I and the strip center voltage U (here maximum standing wave values) this results in the two possible constructions

$$(S^I_{ik})=-(\sqrt{Z_{oi}})(R^I_{ik})^*(R^I_{ik})^{-1}(1/\sqrt{Z_{ok}}), \qquad (5a)$$
$$(S^U_{ik})=-(1/\sqrt{Z_{oi}})(R^U_{ik})^*(R^U_{ik})^{-1}(\sqrt{Z_{ok}}), \qquad (5b)$$

with the left- and right-standing impedance terms denoting diagonal matrices and with

$$R^I_{ik} = I_{ik}\exp(-j\beta_i l_{ik}), \qquad (6a)$$
$$R^U_{ik} = U_{ik}\exp(-j\beta_i l_{ik}). \qquad (6b)$$

Herein, the ports of the microstrip structure are numbered with i and the different resonance experiments with k consecutively. The asterisk * denotes the complex conjugate and the quantity β_i is the phase constant of the microstrip line attached to port i. The matrix elements defined by (6a) and (6b) are computed from the electromagnetic field directly and are independent of the choice of definition of the characteristic impedances Z_{oi}, Z_{ok}. Furthermore it can easily be seen that the condition of resonance of the n-port formulated in terms of the scattering matrices (5a) or (5b), see ref.10, does not depend on the values of Z_{oi}, Z_{ok} employed. Therefore, a criterion for the definition of these cannot be derived from the degree of accuracy to which the resonance condition is satisfied numerically. With the voltage/current definition (1b) the scattering matrices (5a) and (5b) become identical. This is only a conceptual advantage and does not justify the choice of (1b) as a definition. In addition, identical scattering matrices are obtained too if (5a) is used together with (1a) and (5b) together with (1c). It would not be consistent to introduce (1a) into (5b) or (1c) into (5a). The numerical field solutions obtained for microstrip structures do not necessarily result in strictly unitary and reciprocal matrices independent of the definition of the characteristic impedance used. For a two-port scattering matrix assembled according to the rule (5a) for example there results

$$S^I_{12} = +j2I_{11}I_{12}\sin(\phi_{11}-\phi_{12})\sqrt{Z_{01}/Z_{02}}\cdot T, \qquad (8a)$$
$$S^I_{21} = -j2I_{21}I_{22}\sin(\phi_{21}-\phi_{22})\sqrt{Z_{02}/Z_{01}}\cdot T, \qquad (8b)$$

with $\phi_{ik} = \beta_i l_{ik}$.

By construction, the magnitudes of the reflection coefficients of (5a) and (5b) are equal. Also, the phase relationships there are in accordance with the unitariness to be expected as a property of the scattering matrix of a lossless two-port[12]. However, the off-diagonal elements, i.e. the transmission coefficients (8a) and (8b) do not have equal magnitudes by construction (T is a common factor). Instead, numerically computed transmission coefficients are typically seen to exhibit a slight unbalance. This is a function of frequency and of the definition of characteristic impedance chosen and it vanishes as the static case is approached. With the correct choice, the scattering matrices (5a) and (5b) should become unitary and reciprocal within the numerical accuracy achieved even at high frequencies.

RESULTS AND CONCLUSION

Numerical results have been computed for unsymmetrical microstrip gaps and microstrip impedance steps on alumina substrate (thickness h=0.635 mm, dielectric constant 9.7). They confirm what has been outlined before. A possible unbalance in the computed transmission coefficients is tightly related to the ratio Z_{o1}/Z_{o2} in (8a) and (8b). Over a larger scale of frequencies, for example 1-2o GHz, not only the characteristic impedances defined by (1a) to (1c) show a noticeable variation. Also, the relative change with frequency of these increases with the width of a microstrip line as can be seen in fig.2a for some selected cases. Thus, the impedance ratio Z_{o1}/Z_{o2} decreases with frequency if 2 is the wider one of the two microstrip lines. This effect is quite noticeable too and is characterized quantitatively by the function Q in fig.2b.

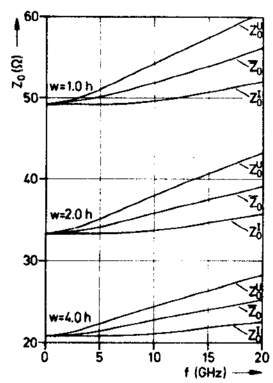

Fig.2a: Z_o as a function of frequency and width

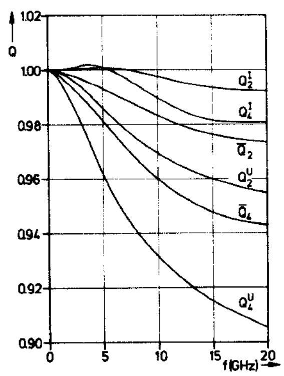

Fig.2b: $Q=(Z_{o1}/Z_{o2})$/(static ratio), the subscript is w_1/w_2

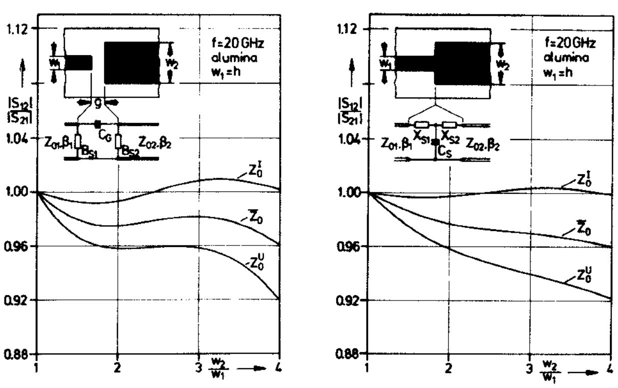

Fig.3: Unbalance of numerically computed transmission coefficients for microstrip gaps and impedance steps

The unbalance itself as emerging typically in the spectral-domain hybrid-mode computation of microstrip configurations is depicted in fig.3. It is obviously in close relation to the function Q of fig.2b. The scattering parameters of fig.3 have been computed according to the formula (5a). It is observed that within the numerical accuracy achieved only the definition (1a) of microstrip characteristic impedance leads to unitary matrices of the considered two-ports, as has been predicted. The same is valid for (1c) in conjunction with (5b). Therefore, for microstrip network concepts using scattering matrices it is essential to include the power P into the definition of characteristic impedance. This is sufficient to describe the magnitudes of wave amplitudes correctly. The phase information is then supplied by the use of quantities like I or U which are directly related to the transverse field. Several tests have been made to check the validity of the computed data. A variety of equivalent circuit data for the unsymmetrical gap is presented in ref.13. In fig.4, some of the computed microstrip impedance step data are compared with the static results of other authors. The agreement is found to be good and the step capacitances of fig.4 are seen to change weakly with frequency up to 2o GHz.

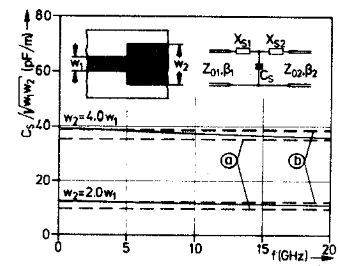

Fig.4: Computed step capacitances in comparison with
(a) - ref. 14 and
(b) - ref. 15

The questions remains which of the quantities I or U is a more accurate representative of the respective transverse field, magnetic or electric at high frequencies. The answer is I which is evident from the weak frequency dependence of the expression (1a). As has been shown this is not a crucial question for the generation of scattering matrices from the electromagnetic field and for CAD concepts based on these. However, it should be considered if equivalent circuits or impedance matrices shall be computed in a way which is physically realistic as far as possible. Also this is often of interest in measurement techniques and associated deembedding problems. Since a physically unique answer cannot be given and the best arguments are in favour of the definition (1a) it is recommended to use this in most microstrip applications.

REFERENCES

1 R.P. Owens, Electr.Letters 12, 1976, 269-27o
2 B. Bianco et al., IEEE Trans.MTT-26, 1978, 182-185
3 W.J. Getsinger, IEEE Trans.MTT-27, 1979, 293
4 F. Arndt, G.U. Paul, IEEE Trans.,ibd., 724-731
5 T.C. Edwards, John Wiley, New York, 1981, Chapt. 4.8
6 Comments to 2, IEEE Trans.MTT-28, 198o, 152
7 J.P. Knorr et al., IEEE Trans.MTT-23, 1975, 725-728
8 R.H. Jansen, IEEE Trans.MTT-26, 1978, 75-82
9 R.H. Jansen, IEE Proc.H, MOA-3, 1979, 14-22
1o R.H. Jansen, IEE Proc.H, 128, 1981 77-86
11 S. Schelkunoff, Bell Syst.Techn.J.17, 1938, 17-48
12 H. Brand, Hirzel Verlag Stuttgart,W.Germany,197o
13 N.H.L. Koster, R.H. Jansen,IEEE Trans.MTT-3o,to be p.
14 P. Benedek, P. Silvester,IEEE Trans.MTT-2o,1972,729
15 B. Easter et al.,radio el.eng.,48,1978, 73-84